

Applications of the derivative operator on some subclasses of analytic functions

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Introduction

We begin by letting $\mathcal{U} = \{z \in \mathbb{C}: |z| < 1\}$ be the open unit disk of the complex plane and \mathcal{A} denote the class of functions of the form

$$f(z) = z + \sum_{k=2}^{\infty} a_k z^k,$$

which are analytic in \mathcal{U} and satisfy the following usual normalization conditions $f(0) = f'(0) - 1 = 0$.

Also, let S be the class of functions $f \in \mathcal{A}$ which are univalent in \mathcal{U} . Given two analytic functions f and g , the subordination between them is written as $f \prec g$ or $f(z) \prec g(z)$, $(z \in \mathcal{U})$. In addition, we say $f(z)$ is subordinate to $g(z)$ if there is a Schwarz function w with $w(z) = 0$, $|w(z)| < 1$, $(z \in \mathcal{U})$ such that $f(z) = g(w(z))$ for all $z \in \mathcal{U}$. Furthermore, if $g(z)$ is univalent in \mathcal{U} , then $f \prec g$ if and only if $f(0) = g(0)$ and $f(\mathcal{U}) \subseteq g(\mathcal{U})$.

In [6], [7], for a function $f \in \mathcal{A}$ and $0 < q < 1$ Jackson defined the q -derivative operator D_q as follows:

$$D_q f(z) = \frac{f(qz) - f(z)}{(q-1)z}, \quad (z \neq 0)$$

and $D_q f(0) = f'(0)$ and $D_q^2 f(z) = D_q(D_q f(z))$. In case $f(z) = z^k$ for k is a positive integer, the q -derivative of $f(z)$ is given by

$$D_q z^k = \frac{z^k - (zq)^k}{z(1-q)} = [k]_q z^{k-1},$$

where $[k]_q$ defined by

$$[k]_q = \frac{1 - q^k}{1 - q}$$

As $q \rightarrow 1^-$ and $k \in \mathbb{N}$, $[k]_q \rightarrow k$

From (1.1) and (1.2) we get that

$$D_q f(z) = 1 + \sum_{k=2}^{\infty} [k]_q a_k z^{k-1}.$$

As a right inverse, Jackson [6] introduced the q -integral

$$\int_0^z f(t) d_q t = z(1-q) \sum_{k=0}^{\infty} q^k f(zq^k)$$

The authors in [1] defined the q - analogue of Ruscheweyh Operator \mathcal{R}_q^λ by

$$\mathcal{R}_q^\lambda f(z) = z + \sum_{k=2}^{\infty} \frac{[k+\lambda-1]_q!}{[\lambda]_q! [k-1]_q!} a_k z^k$$

where $[k]_q!$ defined by :

$$[k]_q! = \begin{cases} [k]_q [k-1]_q \dots [1]_q, & k = 1, 2, \dots \\ 1; & k = 0 \end{cases}$$

All the details about q - calculus used in this paper can be found in [3] and [4].

Also, as $q \rightarrow 1^-$ we have

$$\begin{aligned} \lim_{q \rightarrow 1^-} \mathcal{R}_q^\lambda f(z) &= z + \lim_{q \rightarrow 1^-} \left[\sum_{k=2}^{\infty} \frac{[k+\lambda-1]_q!}{[\lambda]_q! [k-1]_q!} a_k z^k \right] \\ &= z + \sum_{k=2}^{\infty} \frac{(k+\lambda-1)!}{(\lambda)! (k-1)!} a_k z^k \\ &= \mathcal{R}^\lambda f(z) \end{aligned}$$

where $\mathcal{R}^\lambda f(z)$ is Russcheweyh differential operator which was defined in [13] and has been studied by several authors, for example [9] and [15].

The following class of analytic functions can be defined as a result of full utilization of subordination and q -derivative principle.

Definition 1.1 For $\phi \in \mathcal{P}$, $b \in \mathbb{C}^* = \mathbb{C} \setminus \{0\}$, $\beta \geq 0$ and $\lambda > -1$. The function f is said to be in the class $\mathcal{M}_{q,b,\lambda}^\beta(\phi)$ if it is satisfies the condition :

$$1 + \frac{1}{b} \left[\frac{z D_q(\mathcal{R}_q^\lambda f(z)) + \beta z^2 D_q(D_q(\mathcal{R}_q^\lambda f(z)))}{\mathcal{R}_q^\lambda f(z)} - 1 \right] < \phi(z)$$

Remark 1.1 It can be seen that, by specializing the parameters, the class $\mathcal{M}_{q,b,\lambda}^\beta(\phi)$ is reduced to numerous known subclasses of analytic functions, for example:

- If $b = 1$ and $\beta = 0$ we obtain $\mathcal{M}_{q,1,\lambda}^\beta(\phi) \equiv \mathcal{S}_{q,\lambda}^*(\phi)$ (Aldweby and Darus [2]). - If $\lambda = 0$ and $\beta = 0$ we obtain $\mathcal{M}_{q,1,\lambda}^\beta(\phi) \equiv \mathcal{S}_{q,b}(\phi)$ (Seoudy and Aouf [14]).
- If $\lambda = 0$, $\lim_{q \rightarrow 1} \mathcal{M}_{q,b,0}^\beta(\phi) = \mathcal{M}_{\alpha,b}(\phi)$ (Suchithra et al. [16]).
- If $\lambda = 0$ and $\beta = 0$, $\lim_{q \rightarrow 1} \mathcal{M}_{q,b,0}^0(\phi) = \mathcal{S}_b(\phi)$ (Ravichandran et al. [11]).

- If $\lambda = 0$ and $\beta = 0$, $\lim_{q \rightarrow 1} \mathcal{M}_{q,b,0}^0 \left(\frac{1+z}{1-z} \right) = \mathcal{S}^*(b)$ (Nasr and Aouf [10]).
- If $\lambda = 0$ and $\beta = 0$, $\lim_{q \rightarrow 1} \mathcal{M}_{q,1-\eta,0}^0 \left(\frac{1+z}{1-z} \right) = \mathcal{S}^*(\eta)$ (Robertson [12])

In 1933 a study conducted by Fekete and Szegö [5] revealed that the maximum value of $|a_3 - \mu a_2^2|$ as a function of the real parameters μ , for functions belonging to the class \mathcal{S} . Following this, several attempts and researchers solved the Fekete-Szegö problem for various classes of \mathcal{S} class in addition to subclasses of functions in \mathcal{A} . Similar work is shown in [2], [14] and [16]. In this paper, the Fekete-Szegö inequality is obtained for functions in a more general class of $\mathcal{M}_{q,b,\lambda}^\beta(\phi)$ for functions defined above.

In order to prove and validate our results, following preliminary results are required.

4. Preliminary Results

Lemma 2.1 [8] If $p(z) = 1 + c_1 z + c_2 z^2 + \dots \in \mathcal{P}$ of positive real part is in \mathcal{U} and μ a complex number, then

$$|c_2 - \mu c_1^2| \leq 2 \text{ m } \{1; |2\mu - 1|\}.$$

The result is sharp given by

$$p(z) = \frac{1+z}{1-z} \text{ and } p(z) = \frac{1+z^2}{1-z^2}.$$

Lemma 2.2 [8] If $p(z) = 1 + c_1 z + c_2 z^2 + \dots$ is a function with positive real part, then

$$|c_2 - vc_1^2| = \begin{cases} -4v + 2, & \text{if } v \leq 0 \\ 2, & \text{if } 0 \leq v \leq 1 \\ 4v - 2, & \text{if } v \geq 1 \end{cases}$$

5. Main Results

Now is our theorems using similar methods studied by Aldweby and Darus in [2].

Theorem 3.1 Let $\phi(z) = 1 + B_1 z + B_2 z^2 + \dots \in \mathcal{P}$. If f given by (1.1) is in the class $\mathcal{M}_{q,b,\lambda}^\beta(\phi)$ and μ is a complex number, then

$$\begin{aligned} |a_3 - \mu a_2^2| &\leq \frac{[2]_q |bB_1|}{[(1 + [2]_q \beta)[3]_q - 1][\lambda + 1]_q [\lambda + 2]_q} \cdot \text{m } \left\{ 1, \left| \frac{B_2}{B_1} + \right. \right. \\ &\quad \left. \left. \frac{[2]_q [(1 + \beta)[2]_q - 1][\lambda + 1]_q - \mu [(1 + [2]_q \beta)[3]_q - 1][\lambda + 2]_q}{[2]_q [(1 + \beta)[2]_q - 1]^2 [\lambda + 1]_q} bB_1 \right| \right\}. \end{aligned}$$

The result is sharp.

Proof. If $f \in \mathcal{M}_{q,b,\lambda}^\beta(\phi)$, then there is a function $w(z)$ in \mathcal{U} with $w(0) = 0$ and $|w(z)| < 1$ in \mathcal{U} such that

$$1 + \frac{1}{b} \left[\frac{z D_q(\mathcal{R}_q^\lambda f(z)) + \beta z^2 D_q(D_q(\mathcal{R}_q^\lambda f(z)))}{\mathcal{R}_q^\lambda f(z)} - 1 \right] = \phi(w(z)).$$

Define the function $p(z)$ by

$$p(z) = \frac{1 + w(z)}{1 - w(z)} = 1 + d_1 z + d_2 z^2 + \dots$$

Since $w(z)$ is a Schwarz function, immediately $\operatorname{Re}(p(z)) > 0$ and $p(0) = 1$. Therefore,

$$\begin{aligned} \phi(w(z)) &= \phi\left(\frac{p(z) - 1}{p(z) + 1}\right) \\ &= \phi\left(\frac{1}{2} \left[d_1 z + \left(d_2 - \frac{d_1^2}{2}\right) z^2 + \left(d_3 - d_1 d_2 + \frac{d_1^3}{4}\right) z^3 + \dots \right]\right) \\ &= 1 + \frac{1}{2} B_1 d_1 z + \left[\frac{1}{2} B_1 \left(d_2 - \frac{d_1^2}{2}\right) + \frac{1}{4} B_2 d_1^2 \right] z^2 + \dots \end{aligned}$$

since

$$\begin{aligned} 1 + \frac{1}{b} \left[\frac{z D_q(\mathcal{R}_q^\lambda f(z)) + \beta z^2 D_q(D_q(\mathcal{R}_q^\lambda f(z)))}{\mathcal{R}_q^\lambda f(z)} - 1 \right] &= 1 + \frac{[(1 + \beta)[2]_q - 1][\lambda + 1]_q}{b} a_2 z \\ &+ \left[\frac{[(1 + [2]_q \beta)[3]_q - 1][\lambda + 1]_q [\lambda + 2]_q}{[2]_q b} a_3 - \frac{[(1 + \beta)[2]_q - 1][\lambda + 1]_q^2}{b} a_2^2 \right] z^2 + \dots \end{aligned}$$

From (3.2) and (3.3), we obtain

$$\begin{aligned} \frac{[(1 + \beta)[2]_q - 1][\lambda + 1]_q}{b} a_2 &= \frac{B_1 d_1}{2} \\ \frac{[(1 + [2]_q \beta)[3]_q - 1][\lambda + 1]_q [\lambda + 2]_q}{[2]_q b} a_3 - \frac{[(1 + \beta)[2]_q - 1][\lambda + 1]_q^2}{b} a_2^2 &= \frac{1}{2} B_1 \left(d_2 - \frac{d_1^2}{2}\right) + \frac{1}{4} B_2 d_1^2, \end{aligned}$$

or, equivalently,

$$\begin{aligned}
a_2 &= \frac{bB_1d_1}{2[(1+\beta)[2]_q-1][\lambda+1]_q}, \\
&= \frac{[2]_q bB_1 d_2}{2[(1+[2]_q\beta)[3]_q-1][\lambda+1]_q[\lambda+2]_q} + \\
&\quad \frac{[2]_q d_1^2}{4[(1+[2]_q\beta)[3]_q-1][\lambda+1]_q[\lambda+2]_q} \left[\frac{b^2 B_1^2}{[(1+\beta)[2]_q-1]} - b(B_1 - B_2) \right].
\end{aligned}$$

Therefore,

$$a_3 - \mu a_2^2 = \frac{[2]_q bB_1}{2[(1+[2]_q\beta)[3]_q-1][\lambda+1]_q[\lambda+2]_q} (d_2 - \nu d_1^2)$$

where

$$\nu = \frac{1}{2} \left[1 - \frac{B_2}{B_1} - \frac{b[2]_q[(1+\beta)[2]_q-1][\lambda+1]_q - \mu b[(1+[2]_q\beta)[3]_q-1][\lambda+2]_q}{[2]_q[(1+\beta)[2]_q-1]^2[\lambda+1]_q} B_1 \right].$$

Our result now follows by an application of Lemma 2.1. Again by Lemma 2.1, the equality in (3.1) is gained for

$$p(z) = \frac{1+z}{1-z} \quad \text{or} \quad p(z) = \frac{1+z^2}{1-z^2}.$$

Thus the proof of Theorem (3.1) is complete.

Setting $q \rightarrow 1^-$ and $\lambda = 0$ in Theorem 3.1, we obtain the following result

Corollary 3.1 [16] Let $\phi(z) = 1 + B_1 z + B_2 z^2 + \dots \in \mathcal{P}$. If f given by (1.1) is in the class $\mathcal{M}_{\beta,b}(\phi)$ and μ is a complex number, then

$$|a_3 - \mu a_2^2| \leq \frac{|B_1 b|}{2(1+3\beta)} m \left\{ 1, \left| \frac{B_2}{B_1} + \left[\frac{(1+2\beta) - 2\mu(1+3\beta)}{(1+2\beta)^2} \right] bB_1 \right| \right\}.$$

The result is sharp.

Taking $\lambda = 0$ and $\beta = 0$ in Theorem 3.1, we obtain the following result.

Corollary 3.2 [14] Let $\phi(z) = 1 + B_1 z + B_2 z^2 + \dots \in \mathcal{P}$. If f given by (1.1) is in the class $\mathcal{S}_{q,b}(\phi)$ and μ is a complex number, then

$$|a_3 - \mu a_2^2| \leq \frac{|B_1 b|}{[3]_q - 1} m \left\{ 1, \left| \frac{B_2}{B_1} + \left[\frac{([2]_q - 1) - \mu([3]_q - 1)}{([2]_q - 1)^2} \right] bB_1 \right| \right\}.$$

The result is sharp.

By using Lemma 2.2, we have the following theorem.

Theorem 3.2 Let $\phi(z) = 1 + B_1z + B_2z^2 + \dots$ with $B_1 > 0$ and $B_2 \geq 0$. Let

$$\begin{aligned}\rho_1 &= \frac{[2]_q[(1+\beta)[2]_q-1]^2[\lambda+1]_q[bB_1^2+[(1+\beta)[2]_q-1](B_2-B_1)]}{bB_1^2[\lambda+2]_q[(1+\beta)[2]_q-1][(1+[2]_q\beta)[3]_q-1)]}, \\ \rho_2 &= \frac{[2]_q[(1+\beta)[2]_q-1]^2[\lambda+1]_q[bB_1^2+[(1+\beta)[2]_q-1](B_2+B_1)]}{bB_1^2[\lambda+2]_q[(1+\beta)[2]_q-1][(1+[2]_q\beta)[3]_q-1]}.\end{aligned}$$

Let f given by (1.1) be in the class $\mathcal{M}_{q,b,\lambda}^\beta(\phi)$. Then

$$\begin{aligned}|a_3 - \mu a_2^2| &\leq \begin{cases} \frac{[2]_q b B_2}{\gamma} + \frac{[2]_q b B_1^2}{\gamma} \left(\frac{b[2]_q[(1+\beta)[2]_q-1][\lambda+1]_q - \mu b \gamma / [\lambda+1]_q}{[2]_q[(1+\beta)[2]_q-1]^2[\lambda+1]_q} \right), & \text{if } \mu \leq \rho_1; \\ \frac{[2]_q b B_1}{\gamma}, & \text{if } \rho_1 \leq \mu \leq \rho_2 \\ -\frac{[2]_q b B_2}{\gamma} - \frac{[2]_q b B_1^2}{\gamma} \left(\frac{b[2]_q[(1+\beta)[2]_q-1][\lambda+1]_q - \mu b \gamma / [\lambda+1]_q}{[2]_q[(1+\beta)[2]_q-1]^2[\lambda+1]_q} \right), & \text{if } \mu \geq \rho_2, \end{cases} \end{aligned}$$

where $\gamma = [(1+[2]_q\beta)[3]_q-1][\lambda+1]_q[\lambda+2]_q$.

Proof. First, let $\mu \leq \rho_1$

$$\begin{aligned}|a_3 - \mu a_2^2| &\leq \frac{[2]_q b B_1}{2[(1+[2]_q\beta)[3]_q-1][\lambda+1]_q[\lambda+2]_q}[-4\nu+2] \\ &\leq \frac{[2]_q b B_2}{\gamma} + \frac{[2]_q b B_1^2}{\gamma} \left(\frac{b[2]_q[(1+\beta)[2]_q-1][\lambda+1]_q - \mu b \gamma / [\lambda+1]_q}{[2]_q[(1+\beta)[2]_q-1]^2[\lambda+1]_q} \right).\end{aligned}$$

Now, let $\rho_1 \leq \mu \leq \rho_2$, then using the above calculation, we get

$$|a_3 - \mu a_2^2| \leq \frac{[2]_q b B_1}{[(1+[2]_q\beta)[3]_q-1][\lambda+1]_q[\lambda+2]_q}.$$

Finally, if $\mu \geq \rho_2$, then

$$\begin{aligned}|a_3 - \mu a_2^2| &\leq \frac{[2]_q b B_1}{2[(1+[2]_q\beta)[3]_q-1][\lambda+1]_q[\lambda+2]_q}[4\nu-2] \\ &\leq -\frac{[2]_q b B_2}{\gamma} - \frac{[2]_q b B_1^2}{\gamma} \left(\frac{b[2]_q[(1+\beta)[2]_q-1][\lambda+1]_q - \mu b \gamma / [\lambda+1]_q}{[2]_q[(1+\beta)[2]_q-1]^2[\lambda+1]_q} \right),\end{aligned}$$

where $\gamma = [(1+[2]_q\beta)[3]_q-1][\lambda+1]_q[\lambda+2]_q$.

Taking $\beta = 0$ in Theorem 3.2 and knowing that $[n]_q - 1 = q[n-1]_q$, $[n+1]_q = [n]_q + q^n$ and $[n+2]_q = [n]_q + q^n[2]_q$, we obtain the following result

Corollary 3.3 [2] Let $\varphi(z) = 1 + B_1z + B_2z^2 + \dots$ with $B_1 > 0$ and $B_2 \geq 0$. Let

$$\sigma_1 = \frac{([\lambda]_q + q^\lambda)(B_1^2 + q(B_2 - B_1))}{([\lambda]_q + q^\lambda[2]_q)B_1^2}, \quad \sigma_2 = \frac{([\lambda]_q + q^\lambda)(B_1^2 + q(B_2 + B_1))}{([\lambda]_q + q^\lambda[2]_q)B_1^2}.$$

Let f given by (1.1) be in the class $\mathcal{S}_{q,\lambda}^*(\varphi)$. Then

$$\begin{aligned} & |a_3 - \mu a_2^2| \\ & \leq \begin{cases} \frac{B_2}{q[\lambda+1]_q[\lambda+2]_q} + \frac{B_1^2}{q[\lambda+1]_q[\lambda+2]_q} \left(\frac{[\lambda]_q + q^\lambda - ([\lambda]_q + q^\lambda[2]_q)\mu}{q([\lambda]_q + q^\lambda)} \right), & \text{if } \mu \leq \sigma_1; \\ \frac{B_1}{q[\lambda+1]_q[\lambda+2]_q}, & \text{if } \sigma_1 \leq \mu \leq \sigma_2 \\ -\frac{B_2}{q[\lambda+1]_q[\lambda+2]_q} - \frac{B_1^2}{q[\lambda+1]_q[\lambda+2]_q} \left(\frac{[\lambda]_q + q^\lambda - ([\lambda]_q + q^\lambda[2]_q)\mu}{q([\lambda]_q + q^\lambda)} \right), & \text{if } \mu \geq \sigma_2. \end{cases} \end{aligned}$$

6. References

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